The length of a laterally-moving rod

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Einstein used the Lorentz-equations to transform the instantaneous-simultaneous position-coordinates at the beginning (moving-tail) and end (moving-nose) of a rod, within an inertial reference-frame relative to which the rod is moving with a speed v, into the moving inertial reference-frame within which the rod itself is actually stationary; and claimed that such a rod contracts when it is moving. Here, the change in length of a rod passing at speed v, is derived by Lorentz-transforming the stationary position-coordinates of the beginning and end of the rod within the moving inertial reference-frame (within which the rod itself is stationary) into the stationary inertial reference frame (relative to which the rod is moving with speed v): An increase in the length of the moving rod is obtained. It is shown that this length-dilation is demanded by any moving matter-entity in order for this entity to have a de Broglie wavelength.

1. Background and introduction

1.1 Einstein's postulate

There exists no better demonstration of Einstein's genius than his insight in 1905 that the Lorentz-transformation mandates that the speed of light, measured relative to different bodies moving relative to one another, must have the same magnitude $c \cong 3x10^8$ m/s relative to any, each, and all of these bodies [1]. In terms of Galilean terminology, all bodies which are stationary relative to one another, jointly defines an inertial reference-frame (IRF); while all bodies moving with the same velocity **v** relative to these stationary bodies, also define an inertial reference-frame (IRF) within which the latter bodies are stationary. Since there are many bodies moving with many velocities relative to one another, an infinite set of IRF's exists within our Universe.

The relative-motion of different bodies has been visualised in an abstract manner by the motion of different IRF's within each of which there are bodies which are stationary; and where the motion of such an IRF is mathematically modelled in terms of a Cartesian coordinate-system that is moving through Euclidean space. The respective position-coordinates $(x^/,y^/,z^/)$ and (x,y,z) within two inertial reference frames IRF=$K^/$ and IRF=K, passing one another with a relative speed v are compared by assuming that the times on two clocks within IRF=$K^/$ and IRF=K respectively have been synchronized to both read zero when the origins of the coordinate systems coincided. Although the Lorentz transformation from one IRF into the other was known before Einstein postulated that the speed of light must always be the same relative to all bodies in the universe, no matter with what speed such a body moves relative to other bodies, it is at present accepted that this postulate is responsible for the physics-reality which demands the validity of the Lorentz-equations.

1.2 The Lorentz-transformation

A primary event is defined as an event within an IRF which will occur at the same position-coordinates within this IRF if it were to occur at a later or earlier time [2]. If it has to occur at different position coordinates within an IRF at different times, it is not a primary event within this IRF. In other words to be a primary event, the cause of the event must be stationary within the IRF within which the event occurs.

The Lorentz-transformation (LT) for a primary event at time $t^/$ and position $(x^/,y^/,z^/)$ within IRF=$K^/$ into IRF=K, follows as:

$$x_{LT} = \frac{x^/ + vt^/}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma(x^/ + vt^/) \tag{1a}$$

$$y_{LT} = y^/ \tag{1b}$$

$$z_{LT} = z^/ \tag{1c}$$

And

$$t_{LT} = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\left(t' + \frac{vx'}{c^2}\right) \qquad (1d)$$

In order to make physics-sense, only a primary event within IRF=K$^{/}$ can be transformed from IRF=K$^{/}$ into IRF=K by means of these equations.

If a primary event occurs at time t and a position (x,y,z) within IRF=K, it can be transformed into IRF=K$^{/}$ by means of the reverse Lorentz-transformation, which is given by the following equations:

$$x'_{LT} = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma(x - vt) \qquad (2a)$$

$$y'_{LT} = y \qquad (2b)$$

$$z'_{LT} = z \qquad (2c)$$

And

$$t'_{LT} = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\left(t - \frac{vx}{c^2}\right) \qquad (2d)$$

It does not make physics-sense when a primary event within IRF=K$^{/}$, after being Lorentz transformed into IRF=K, is transformed back into IRF=K$^{/}$ by means of this reverse Lorentz transformation [2]. Only an event which is a primary event within IRF=K can be transformed into IRF=K$^{/}$ by means of the reverse Lorentz transformation.

It has been found that, contrary to what has been believed for more than 100 years, the time t_{LT} for a LT-event within IRF=K is not simultaneous with the time $t^{/}$ within IRF=K$^{/}$, when the primary event occurs within IRF=K$^{/}$ [2]. When the primary event occurs within IRF=K$^{/}$ at the time $t^{/}$, the clock within IRF=K simultaneously shows the time t where $t=t^{/}$. Similarly, when the transformed event is observed within IRF=K at the LT time t_{LT}, at the position (x_{LT},y_{LT},z_{LT}), the time on the clock within IRF=K$^{/}$ shows simultaneously the same time t_{LT}.

Furthermore, owing to the non-simultaneous times for a primary event within IRF=K$^{/}$ after being Lorentz transformed into IRF=K, there is not any Lorentz-Fitzgerald length-contraction. In fact, there is just the opposite, namely a length-dilation. Einstein, however, derived that a rod (or meter stick) which is stationary along the $x^{/}$-direction within IRF=K$^{/}$ will contract when it is observed within IRF=K while passing by with a speed v. In doing so, Einstein transformed non-primary events within IRF=K into IRF=K$^{/}$ by using the reverse Lorentz-transformation. Here Einstein's derivation is revisited, analysed, modified and discussed.

2. A passing rod

2.1 Einstein's derivation

For a stationary rod of length $L^{/}$ within IRF=$K^{/}$, which moves past with a speed v relative to IRF=K, Einstein deduced a contraction in the rod within IRF=K so that it has a length $L<L^{/}$. He motivated this derivation as follows [3]: "*I place a meter-rod* (Einstein chose the length of the rod as unity: In the present case the rod will be assumed to have a length $L^{/}$) *in the $x^{/}$-axis of $K^{/}$ in such a manner that one end (the beginning) coincides with the point $x^{/}$=0, whilst the other end (the end of the rod) coincides with the point $x^{/}=L^{/}$. What is the length of the meter-rod relatively to the system K? In order to learn this, we need only ask where the beginning of the rod and the end of the rod lie with respect to K at a particular time t of the system K. By means of the first equation of the Lorentz-transformation the values of these two points at the time t=0 can be shown to be:*

$$\text{* (beginning of the rod)} = 0\sqrt{1-\frac{v^2}{c^2}}$$

$$\text{*(end of the rod)} = L^{/}\sqrt{1-\frac{v^2}{c^2}}$$

the distance between the points being $L^{/}$. But the meter-rod is moving with the velocity v relative to K. It therefore follows that the length of a rigid meter-rod moving in the direction of its length with a velocity v, is $\sqrt{1-v^2/c^2}$ *of a metre* (i.e. of the length $L^{/}$). *The rigid rod is thus shorter when in motion than when at rest….*"

Einstein's derivation rests on the inherent assumption that the beginning of the rod and the end of the rod are simultaneously present within the IRF=K. To emphasize here that he made this assumption, we will repeat part of the quote above: "*What is the length of the meter-rod relatively to the system K? In order to learn this, we need only ask where the beginning of the rod and the end of the rod lie with respect to K at a particular time t of the system K*".

Einstein then transformed the beginning (x_b=0) and end (x_e=L) coordinates of this supposedly, instantaneous length L within IRF=K from IRF=K into IRF=$K^{/}$, by using the reverse Lorentz-tranformation: But, since the front and end positions of the rod change with time within IRF=K, their instantaneous positions are not primary events within IRF=K, and can therefore not be transformed into IRF=$K^{/}$ by means of the reverse Lorentz transformation. Even if this would have been physically allowed, these reverse-transformed coordinates of the beginning and end of the rod cannot be simultaneous within IRF=$K^{/}$. Thus, they cannot relate to the actual length $L^{/}$ of the rod within IRF=$K^{/}$ whose front and end coordinates are simultaneously always the same within IRF=$K^{/}$.

1.2 Corrected derivation

The Lorentz-transformed simultaneous-coordinates of two stationary event-positions spaced any distance apart within any IRF, cannot be simultaneous in any other IRF passing by: This would violate the relativistic non-simultaneity of simultaneous events. Since the rod $L^{/}$ is stationary within IRF=$K^{/}$, its beginning ($x_b^{/} = 0$) and end ($x_e^{/} = L^{/}$) coordinates are at any instant in time simultaneously always the same within IRF=$K^{/}$: These positions are thus primary events within IRF=$K^{/}$: and therefore Einstein should have used the Lorentz-transformation (Eq. 1) from IRF=$K^{/}$ into IRF=K: NOT the reverse transformation (Eq. 2). According to Eq. 1a he should have set $x^{/} = x_e^{/} = L^{/}$ and $x_{LT} = x_{LTe} = L$ in order to obtain that:

$$L = \frac{L^{/}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma L^{/} \tag{3}$$

Einstein should thus have found that the transformed length of the rod becomes longer within IRF=K; not shorter!

Furthermore, Einstein should have used the whole Lorentz-transformation which gives that the coordinate x=L at the end of the rod is **not** present within IRF=K at the same time t=0 at which the coordinate x=0 is present at the beginning of the rod. Eq. 1d clearly demands that the coordinate x=L can only be present at a later time $\gamma(vL^{/})/c^2$; which is larger than t=0.

In order to actually derive where the positions of the beginning of the rod and the end of the rod are simultaneously at a particular instant in time within IRF=K, Einstein should have used the coincident coordinates at any time t, given in his example at t=0, for the beginning of the rod by $x_b = x_b^{/} = 0$ and for the end of the rod by $x_e = x_e^{/} = L^{/}$: When subtracting these position-coordinates, at say time t=0, the same instantaneous length for the rod is obtained within IRF=K than the actual length of the rod is within IRF=$K^{/}$.

Thus, at any single instant in time these positions are simultaneous-instantaneous exactly $L^{/}$ apart within **both** IRF=$K^{/}$ and IRF=K; even though, owing to the Lorentz-transformation, an observer at the origin 0 within IRF=K cannot see this instantaneous length [2]. According to the Lorentz-transformation such an observer must see a longer length, and, in addition, a change in time along this length.

But the equation for the increased length L does not contain any mathematical terms that relate to the latter change in time along the rod: The length L is independent of time. What is the meaning of the Lorentz-transformed length L>$L^{/}$ of the moving rod within IRF=K? Could the rod have a different length if its beginning within IRF=$K^{/}$ has not been chosen to be at the origin $x^{/}$=0, and if the Lorentz transformation was not done at the time $t^{/}$=0?

3. Transforming the rod at any position and instant in time

If the LT-transformed length L is a real-physical increase in the stationary length $L^{/}$, it should not change when observed from different positions withinh IRF=K: i.e. the same length L must be obtained when LT-transforming a rod with length $L^{/}$ from a beginning coordinate $x_b^{/} \neq 0$ and at at any time $t^{/} \neq 0$.

Choosing the position of the beginning of the rod within IRF=$K^{/}$ to be any coordinate $x_b^{/}$, so that the coordinate at the end of the rod must be $x_e^{/} = x_b^{/} + L^{/}$, the Lorentz-transformed position-coordinates along the x-direction at any time $t^{/}$ are:

$$x_{LTb} = \frac{x_b^{/} + vt^{/}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma(x_b^{/} + vt^{/}) \qquad (4a)$$

And:

$$x_{LTe} = \frac{x_e^{/} + vt^{/}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma(x_b^{/} + L^{/} + vt^{/}) \qquad (4b)$$

The Lorentz-transformed (LT) length L is obtained as:

$$L = x_{LTe} - x_{LTb} = \gamma L^{/} = \frac{L^{/}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (5)$$

Thus, the rod can be at any position within IRF=$K^{/}$ and the transformation can be done at any instant in time $t^{/}$ to obtain the SAME length L as in Eq. 3.

The time t_{LTb} when the beginning of the rod is at the coordinate $x_{LTb,}$ and the time t_{LTe} when the end of the rod is at position x_{LTe}, are respectivly given by:

$$t_{LTb} = \frac{t^{/} + \frac{v}{c^2} x_b^{/}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\left(t^{/} + \frac{v}{c^2} x_b^{/}\right) \qquad (6a)$$

And:

$$t_{LTe} = \frac{t^{/} + \frac{v}{c^2} x_e^{/}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\left(t^{/} + \frac{v}{c^2}\left(x_b^{/} + L^{/}\right)\right) \qquad (6b)$$

Thus, the time difference ΔT_{rod} between the position-coordinates at the beginning and the end of the rod is:

$$\Delta T_{rod} = t_{LTe} - t_{LTb} = \left(\frac{v}{c^2}\right)\gamma L' = \left(\frac{v}{c}\right)\left(\frac{L}{c}\right) \quad (7)$$

The time difference also does not change with time t: It remains a constant value which is proportional to the LT length L. The length L, and thus also the time difference ΔT_{rod}, are functions of only the stationary length L' and the speed v of the rod: i.e. Both are independent of the time t' on all the clocks within IRF=K' within which the rod of lenghth L' is stationary and the synchronous time $t=t'$ within IRF=K relative to which the rod is moving with a speed v.

Assume now that the end position of the rod L' is at the coordinate $x'_e = 0$ and the beginning thus at $x'_b = -L'$: The LT position-coordinates at time t' are thus:

$$x_{LTb} = \frac{x'_b + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma(-L' + vt') \quad (8a)$$

And:

$$x_{LTe} = \frac{x'_e + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma(0 + vt') \quad (8b)$$

By subtracting x_{LTb} from x_{LTe}, one again obtains the relationship given by Eq. 3. The corresponding time coordinates t_{LTb} and t_{LTe} are:

$$t_{LTb} = \frac{t' + \frac{v}{c^2}x'_b}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\left(t' - \frac{v}{c^2}L'\right) \quad (9a)$$

And:

$$t_{LTe} = \frac{t' + \frac{v}{c^2}x'_e}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma t' \quad (9b)$$

By subtracting t_{LTb} from t_{LTe} one again obtains the time difference given by Eq. 7. Thus whether the rod is approaching the origin 0 within IRF=K, or receding from this origin, it has the same time-independent length L within IRF=K.

Now consider an observer M within the IRF=K who has a stop-watch: When the end of the rod (nose) at $x'_e = 0$ passes the observer at the origin of IRF=K, the observer starts the stop-watch. After a time t the distance between the origins must be vt. Consider the synchronous time $t_R = t'_R$ on the clocks within IRF=K and IRF=K' at which the distance between the origins is $L^{/} = vt_R = vt_R^{/}$ [2]: The LT coordinate-positions of the rod are at this instant in time, according to Eq. 8:

$$x_{LTb} = \gamma(-L^{/} + vt_R^{/}) = 0 \qquad (10a)$$

Exactly what one expects that it should be after the time $t_R^{/} = L^{/}/v$. But:

$$x_{LTe} = \gamma(0 + vt_R^{/}) = \gamma L^{/} \qquad (10b)$$

Which is again the same as Eq. 3. Although M's stopwatch proves that the distance between the origins must be equal to the stationary length $L^{/}$ of the rod, the LT-length of the rod is longer than $L^{/}$! Thus the instantaneous length of the rod remains $L^{/}$ within both IRF=$K^{/}$ and IRK=K, but the relativistic length, which determines the physics within IRF=K, is L.

One is thus forced to accept that in the case of a passing rod the permanent length of the rod within IRF=K must be $L = \gamma L^{/}$, AND also that **within the rod** there is a permanent time difference ΔT_{rod} between the beginning and the end of the rod. Time increases with length from the beginning of the rod to the end of the rod.

4. Mass-energy of a moving rod

Even though time increases along the rod of lenghth L, the deduction that L is a constant physically-real length of the passing rod within IRF=K, is supported by the fact that, according to Einstein's famous formula $E=mc^2$, the rest mass (say m_0) of the rod must increase to add dynamic-mass when it moves at a speed v, so that the total mass becomes m; where one has for m in terms of the rest-mass m_0 that:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (11)$$

If the rod has a cross-sectional area S, its rest-mass volume must be $SL^{/}$, and its density $\rho_{rod} = m_0/(SL^{/})$: Assuming that the increase in length, given by Eq. 2a, is a real increase in matter-energy, the rod's increase in mass, given by Eq. 11, mandates that the mass-density of the rod remains ρ_{rod} for any speed v: i.e.

$$\rho_{rod} = \frac{m_0}{SL^{/}} = \frac{m}{SL} \qquad (12)$$

This seems to be a reasonable result. If, in contrast, the rod had actually contracted in length as Einstein had argued, its matter-energy-density would have had to increase to accommodate this increase in mass. It is more reasonable to assume that if the mass-energy density has a certain value when the rod is stationary, it must also have the same density when the rod moves and becomes longer.

When the rod is stationary with length $L^{/}$, the constituents of the rod are atoms bonded by electrons: Does this mean that a moving rod, which enlongates, consists of more atoms and more electrons than it does when it is stationary? One expects that when a single atom is moving, this atom should also become longer in the direction along which it moves to accommodate its own increase in mass-energy: This does not necessarily mean that such a single atom must sprout extra atoms to form a row of atoms which is moving with the speed v. If this could happen, it would require that the moving atom must increase its mass-energy by "quantum-steps" when increasing its speed! One, however, expects that the increase in mass-energy should be continuous for any body with mass when the speed of the atom increases continuously. This means that the increased energy must be continuoiusly distributed within the increased volume of the moving matter-entity.

5. A solitary moving electron

5.1 Length-increase

An electron's mass-energy must also increase with speed: Therefore, one expects that an electron should also increase in length along the direction in which it is moving. It is, also in this case, unlikely that it will sprout extra electrons to increase it's amount of matter-energy by forming a string of electrons: The latter scenario would require an increase in charge, which has not been observed for fast-moving electrons. Furthermore, the electrons forming such a string will explode away from one another.

It seems compelling to conclude that it must be the actual matter-energy constituting the single electron that increases. Since an electron has not been found (so far) to be divisable into smaller separate components, and since the electron's volume is expected to increase when its speed increases (owing to its concomitant length increase), this matter-energy must be continuously distributed within a confined space that delineates the size of the electron: Thus, there must exist distributed mass-energy within the volume of the electron. If this deduction is correct, a moving electron must be a moving, distributed energy-field in its own right. And since a moving energy-field is a wave, it implies that a moving electron must be an actual wave; and nothing else but an actual wave.

5.2 Time-difference

The time difference across a rod given by Eq. 6, only applies when there is a stationary matter-entity with lenghth $L^{/}$ within a moving IRF=$K^{/}$ passing by with a speed v within IRF=K. If the Lorentz-transformed increased length L is real, the time-difference might, and probably does relate to a property of the matter-entity which changes when it moves.

Since a moving coherent-wave has a changing phase-angle at every point along the length of the wave, one might venture to assign a concomitant phase-time which changes along the length of the wave. Thus the change in time along the increased length might be further evidence that the free motion of any matter-entity is always nothing else than coherent wave-motion.

The simplest object with mass is obviously an electron: Assuming that a stationary electron has a spherical shape with diameter $L_e^{/}$, an electron moving with speed v (and thus momentum $p_e=m_e v$; where m_e is the sum of the rest-mass and dynamic mass) should have a length L_e given by Eq. 4. Thus, if it has a wavelength λ_e the number n of wavelenghths within its length L_e must be given by:

$$n = \frac{L_e}{\lambda_e} \qquad (13)$$

If the wave has a frequency ν_e the difference in phase time ΔT_e across the length of the electron is given by:

$$\Delta T_e = \frac{n}{\nu_e} = \frac{L_e}{\nu_e \lambda_e} \qquad (14)$$

Setting L equal to L_e within Eq. 7, and equating ΔT_{rod} with ΔT_e, give:

$$\lambda_e \nu_e = \frac{c^2}{v} = \frac{m_e c^2}{m_e v} \qquad (15)$$

If one now replaces $m_e c^2$ with the Planck formula $h\nu_e$, Eq. 15 becomes:

$$\lambda_e = \frac{h}{m_e v} = \frac{h}{p} \qquad (16)$$

Which is de Broglie's formula for the wavelength of an electron-wave.

6. Discussion

In view of the derivations above, the concept of "wave-particle" duality is suspect: The results above imply that moving matter consists of electromagnetic-energy which moves at a speed that is less than the speed of light c. Since the distributed electromagnetic-energy (which is the electron) is mass-energy, the electron-wave must have a centre-of-mass which moves like a "point"-particle. Thus, it can be argued that the wave and "particle" behaviours are not two mutually exclusive attributes of an electron which are "complentary": Both behaviours are a direct result of the fact that the electron itself is electromagnetic field-energy and that its intensity relates to this field-energy and not to a "probability-distribution".

Since there are many IRF's within which the electron simultaneously move with different speeds, this demands that the electron simultaneously consists of different sizes and shapes to accomodate different amounts of mass-energy within these different IRF's. It is thus in principle possible that within an IRF, relative to which an electron moves with a speed near the speed of light, the electron-field might be able to stretch across parsecs; while, in contrast, the same electron might only be a few μm long within another IRF relative to which it is moving very slowly. This must mean that when a fast-moving electron is slowed down, its wave intensity must collapse into a smaller volume. In the case where the stopping of the electron is near-instantaneous, the collapse of the wave's volume must also be near-instantaeous. The speed of collapse is not limited by the speed of light.

One thus expects that when an electron impinges at high speed into a material, it will have a long length along its direction of motion before interacting with the material. When entering the material two types of interactions are possible:

(1) If the electron is rapidly slowed down, it will collapse into a smaller volume and collide like a localised entity with a center-of-mass (a "particle").

(2) If the atoms within the material form a suitable periodic array, the electron-wave might rather difffract. Nonetheless, in both cases the electron IS and REMAINS a single holistic-wave with its intensity equal to its distributed mass-energy.

7. Conclusion

It is compelling to conclude that Einstein's derivation that a moving body with mass will contract in length is not correct. The need for length-contraction has been removed by Einstein's own postulate that the speed of light must have the same value c relative to any moving body. Before this postulate, the Lorentz-Fitzgerald contraction was required to derive the Lorentz transformation by combining this contraction with the Galilean transformation. Since this contraction is not required when deriving the Lorentz-transformation in terms of the constancy of the speed of light, it has become irrelevant.

The derivation above implies that Einstein's Special Theory of Relativity in essence predicted de Broglie's wavelength more than two decades before de Broglie postulated this wavelength. It also implies that moving matter consists of electromagnetic-energy which moves at a speed that is less than the speed of light c. This in turn implies that matter, which is stationary within an inertial reference frame, might be nothing else but a stationary electromagnetic field; which moves like an electromagnetic-wave within all the other inertial reference frames within which the matter-entity is not stationary.

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